2.21. Cone-Surface Source.[26]—The directional characteristics[27] of a paper or felted paper cone used in the direct radiator-type loudspeaker may be predicted theoretically from the dimensions and shape of the cone and the velocity of sound propagation in the material. For this type of analysis the cone is divided into a number of ring-type radiators as shown in Fig. 2.24. The dimension of the ring along the cone should be a small

[26] Carlisle, R. W., *Jour. Acous. Soc. Amer.*, Vol. 15, No. 1, p. 44, 1943.

[27] The analysis in this section assumes that there is no reflected wave at the outer boundary. In order to obtain a uniform response frequency characteristic the reflected wave must be small. If the reflected wave is small, the effect upon the directional pattern may be neglected.

fraction of the wavelength of sound in the paper. The output of the cone at any angle is the vector sum of the vectors A_0, A_1, A_2 ... A_n where the A's are the amplitudes of the individual rings.

The phase angle of the amplitude of the first ring is

$$\theta_0 = 0 \qquad\qquad 2.36$$

The phase angle of the amplitude of the second ring is

$$\theta_1 = 2\pi\left(\frac{d_1}{\lambda_A} - \frac{D_1}{\lambda_P}\right)\cos\alpha \qquad\qquad 2.37$$

The phase angle of the amplitude of the third ring is

$$\theta_2 = 2\pi\left(\frac{d_1 + d_2}{\lambda_A} - \frac{D_1 + D_2}{\lambda_P}\right)\cos\alpha \qquad\qquad 2.38$$
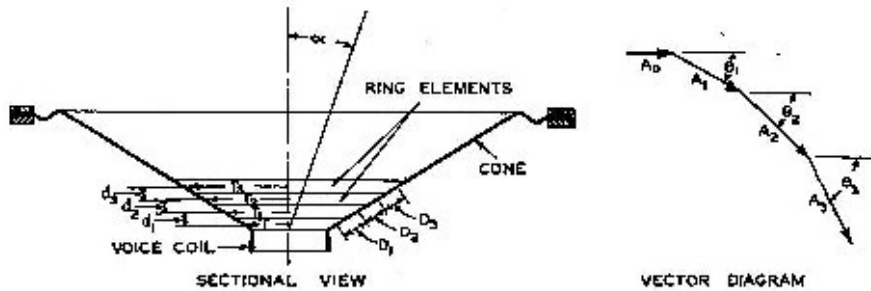

Fig. 2.24. Geometry for obtaining the directional pattern of a cone-type radiator.

The phase angle of the amplitude of the nth ring is

$$\theta_n = 2\pi\left(\frac{d_1 + d_2 \ldots d_n}{\lambda_A} - \frac{D_1 + D_2 \ldots D_n}{\lambda_P}\right)\cos\alpha \qquad\qquad 2.39$$

where $d_1, d_2, \ldots$ = axial distances shown in Fig. 2.24 in centimeters, and
$D_1, D_2, \ldots$ = distances along the cone shown in Fig. 2.24 in centimeters,
λ_A = wavelength of sound in air, in centimeters,
λ_P = wavelength of the sound in the paper cone, in centimeters, and
α = angle between the axis of the cone and the line joining the observation point and the center of the first ring.

The relative amplitude of the vector A_n is given by

$$A_n = 2\pi r_n D_n J_0\left(\frac{2\pi r_n}{\lambda_A}\sin\alpha\right) \qquad\qquad 2.40$$

where r_n = radius of the nth ring, in centimeters,
D_n = width of the nth ring along the cone, in centimeters,
λ_A = wavelength of sound in air, in centimeters,
α = angle between the axis of the cone and the line joining the observation point and the center of the cone, and
J_0 = Bessel function of zero order.

The directional characteristic of the cone is

$$R_a = \frac{\sum\limits_{K=0}^{K=n} A_K \cos \theta_K - j \sum\limits_{K=0}^{K=n} A_K \sin \theta_K}{\sum\limits_{K=0}^{K=n} A_K}$$ 2.41

where R_a = ratio of the pressure for an angle α to the pressure for an angle $\alpha = 0$.

A consideration of equation 2.41 shows that the directional pattern is a function of the frequency and becomes sharper as the frequency increases. For a particular frequency, cone angle, and material the directional patterns are practically similar for the same ratio of cone diameter to wavelength. For a particular frequency and the same cone material the directional pattern becomes broader as the cone angle is made larger. For a particular frequency and cone angle the directional pattern becomes broader as the velocity of propagation in the material decreases (see Sec. 6.2).